Parallel and Distributed Computing

Alberto Paoluzzi – Lecture 21

Mon 4-05-2022

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Parallel and Distributed Computing

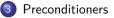
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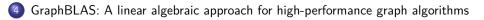
from Templates for the Solution of Linear Sustems: Building Blocks for Iterative Methoda



Why Use Templates in Linear Algebra?







Section 1

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Second, templates exploit the expertise of two distinct groups

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And third, templates are not language specific

• Rather, they are displayed in an Algol-like structure

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Section 2

Iterative Methods

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The nonstationary methods we present are based on the idea of sequences of orthogonal vectors

• (An exception is the Chebyshev iteration method, which is based on orthogonal polynomials.)

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Nonstationary iterative method: Iterative method that has iteration-dependent coefficients

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- Dense matrix: Matrix for which the number of zero elements is too small to warrant specialized algorithms
- Sparse matrix: Matrix for which the number of zero elements is large enough that algorithms avoiding operations on zero elements pay off
- Matrices derived from partial differential equations typically have a number of nonzero elements that is proportional to the matrix size, while the total number of matrix elements is the square of the matrix size.

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- The transformation matrix is called a preconditioner
- A good preconditioner improves the convergence of the iterative method, sufficiently to overcome the extra cost of constructing and applying the pre-conditioner
- Indeed, without a preconditioner the iterative method may even fail to converge.

• The Jacobi Method

- The Jacobi Method
- The Gauss-Seidel Method

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- The Successive Overrelaxation Method

- The Jacobi Method
- The Gauss-Seidel Method
- The Successive Overrelaxation Method
- The Symmetric Successive Overrelaxation Method

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There are many methods for storing the data

• Here we will discuss Compressed Row and Column Storage, Block Compressed Row Storage, Diagonal Storage, Jagged Diagonal Storage, and Skyline Storage.

The efficiency of any of the iterative methods considered in previous sections is determined primarily by the performance of the matrix-vector product and the preconditioner solve, and therefore on the storage scheme used for the matrix and the preconditioner

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- Since iterative methods are typically used on sparse matrices, we will review here a number of sparse storage formats
- Often, the storage scheme used arises naturally from the specific application problem.

Storage scheme:

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The way elements of a matrix are stored in the memory of a computer

- For dense matrices, this can be the decision to store rows or columns consecutively
- For sparse matrices, common storage schemes avoid storing zero elements; as a result they involve integer data describing where the stored elements fit into the global matrix.

• Compressed Row Storage (CRS)

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- Skyline Storage (SKS)

Nonstationary Iterative Methods

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- Typically, constants are computed by taking inner products of residuals or other vectors arising from the iterative method.

Section 3

Preconditioners

Preconditioners

The convergence rate of iterative methods depends on spectral properties of the coefficient matrix

• Hence one may attempt to transform the linear system into one that is equivalent in the sense that it has the same solution, but that has more favorable spectral properties

For instance, if a matrix approximates the coefficient matrix in some way, the transformed system has the same solution as the original system, but the spectral properties of its coefficient matrix may be more favorable.

Preconditioners

The convergence rate of iterative methods depends on spectral properties of the coefficient matrix

- Hence one may attempt to transform the linear system into one that is equivalent in the sense that it has the same solution, but that has more favorable spectral properties
- A preconditioner is a matrix that effects such a transformation.

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Section 4

GraphBLAS: A linear algebraic approach for high-performance graph algorithms

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The GraphBLAS

The GraphBLAS Forum is an open effort to define standard building blocks for graph algorithms in the language of linear algebra.

https://graphblas.org

- A key insight behind this work is that when a graph is represented by a sparse incidence or adjacency matrix, sparse matrix-vector multiplication is a step of breadth first search
 - By generalizing the pair of scalar operations involved in the linear algebra computations to define a semiring, we can extend the range of these primitives to support a wide range of parallel graph algorithms.

GraphBLAS lecture

GraphBLAS: A linear algebraic approach for high-performance graph algorithms